Cumulative Interpretations and the Need of the Plural Projection[*]

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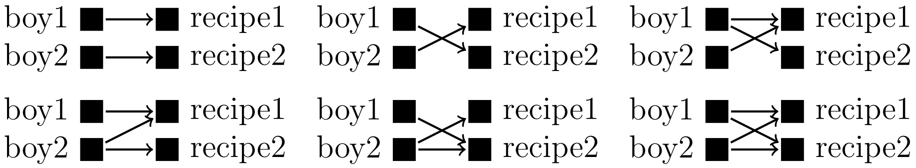
1 Introduction.

Cumulative interpretations: Sentences with two plurals like (1) often have 'weak' truth conditions (2) (e.g., Kroch 1974, Scha 1981).

(1) [The two boys]$_{PL1}$ typed [the two recipes]$_{PL2}$

(2) **One interpretation of (1): (1) is true if at least one of the following scenarios is true.**



(→ := *type*-relation)

Such a weak interpretation is known as the *cumulative interpretation* (e.g., Beck and Sauerland 2000), and its truth condition can be schematized as follows (A and B are plurals, and R is a binary relation).

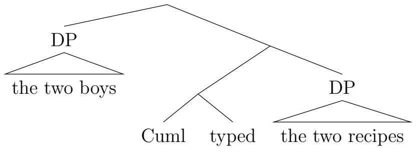
(3) a. A R B
b. 1 iff $\forall x \in S_A(\exists y \in S_B(R(y)(x)=1)) \wedge \forall y \in S_B(\exists x \in S_A(R(y)(x)=1))$
where S_A, S_B are sets of objects that $[\![A]\!]$ and $[\![B]\!]$ consist of intuitively.
(Schmitt 2019, 7)

A source of cumulative interpretations: Such weak truth conditions are attributed to a *Cuml* operator (e.g., Beck and Sauerland 2000) (4).

(4) Cuml (R)(B)(A) = 1 iff $\forall x \in S_A(\exists y \in S_B(R(y)(x)=1)) \wedge \forall y \in S_B(\exists x \in S_A(R(y)(x)=1))$
where S_A, S_B are sets of objects that $[\![A]\!]$ and $[\![B]\!]$ consist of intuitively.
(adapted from Beck and Sauerland 2000, 351)

The Cuml operator captures the cumulative interpretation of (1) by assuming the LF in (5).

(5)



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[*] I would like to thank Bernhard Schwarz and Michael Wagner for their insightful comments and questions on this project. I also wish to thank Alan Bale, Christopher Estrada, Jonathan Palucci, Viola Schmitt, Yale Sharvit, and Andrew Suson as well as participants in the McGill syntax/semantics meeting and TOM13 for providing their acceptability judgments and/or having useful discussions about cumulativity with me at different stages of this project.

$$\llbracket \text{TP} \rrbracket = 1 \text{ iff } \forall x{\in}\{\text{boy1,boy2}\}(\exists y{\in}\{\text{recipe1,recipe2}\}(\textsc{typed}(y)(x){=}1)) \wedge$$
$$\forall y{\in}\{\text{recipe1,recipe2}\}(\exists x{\in}\{\text{boy1,boy2}\}(\textsc{typed}(y)(x){=}1))$$

The Cuml operator undergenerates: Schmitt (2019) observes the Cuml does not derive cumulations in so-called *flattening examples.*

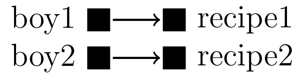
(6) **PL flattening**
[The two boys]$_{PL1}$ made Abe [type [these two recipes]$_{PL2}$ and create this blog]$_{PL3}$.
(adapted from Schmitt 2019, 27)

- **Truth condition**: There is a cumulative relation between the following two sets.
  {boy1,boy2}
  {MADE-ABE-TYPE-RECIPE1, MADE-ABE-TYPE-RECIPE2, MADE-ABE-CREATE-BLOG}

One of the scenarios for (6) to be true is the following:.

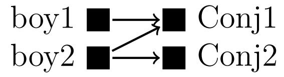
(7) **Scenario in which (6) is true**
Boy1 made Abe type recipe1. Boy2 made Abe type recipe2 and create this blog.

Cumulation between PL1 and PL2



(→ := *'made Abe type'*-relation)

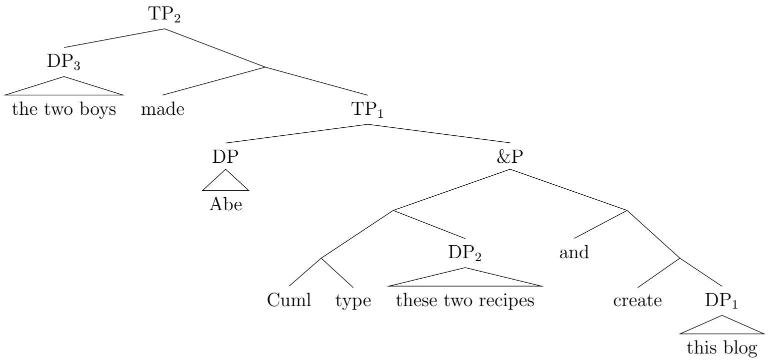
Cumulation between PL1 and PL3



(→ := *'made Abe'*-relation)

The Cuml operator does not enable (6) to be true in (7); There are no predicates the Cuml can take to capture the cumulative relations in (7). For example, the Cuml takes *type* in (8), but the LF does not derive the cumulative interpretations.

(8)



Schmitt's (2019) solution: Propose a new source of cumulations called the *plural projection.* It consists of assumptions about plurals and a new compositional rule, which enable to capture flattening examples.

- The plural projection effectively enables to derive the following set of functions, and let it cumulatively compose with {boy1,boy2}.
  {MADE-ABE-TYPE-RECIPE1, MADE-ABE-TYPE-RECIPE2, MADE-ABE-CREATE-BLOG}

Alternative solution: As hinted by Schmitt (2020), however, I observe that another source of cumulations known as the *lexical source* (e.g., Krifka 1989) can capture flattening examples using an event semantics.

- The lexical source effectively derives a set of events {e_1,e_2,e_3}, which ends up having a cumulative *made*-relation with {boy1,boy2}.

  e$_1$: Abe typed recipe1.

  e$_2$: Abe typed recipe2.

  e$_3$: Abe created blog.

So both the plural projection and lexical source can derive cumulations in flattening examples. But do flattening examples require *both* of the sources of cumulations?

Lexical source is independently needed: Based on a version of flattening examples like (9), I claim that flattening examples require the lexical source.

(9) **SG flattening**

[Assumption: The referent of *the ramen recipe* consists of two recipes: a recipe about how to make noodle, and a recipe about how to make broth.]

[The two boys]$_{PL1}$ made Abe [type **[the ramen recipe]**$_{SG}$ and create this blog]$_{PL2}$.

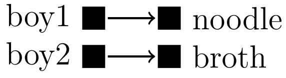
- **Truth condition**: There is a cumulative relation between the following two sets.
  {boy1,boy2}
  {MADE-ABE-TYPE-NOODLE, MADE-ABE-TYPE-BROTH, MADE-ABE-CREATE-BLOG}
  (‘noodle’ and ‘broth’ refer to the noodle broth recipe.)
- Sentence (9) with the given assumption is true in the same scenario as (6) if *recipe1* and *recipe2* in (6) are equivalent to *noodle* and *broth* in (9), respectively.

While the lexical source enables (9) to be true in the scenario in (10), the plural projection doesn't.

(10) **Scenario in which (9) is true**

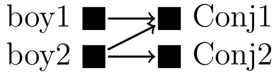
There is a ramen recipe that consists of two recipes: a recipe about how to make noodles, and a recipe about how to make broth. Boy1 made Abe type the noodle recipe. Boy2 made Abe type the broth recipe, and create a blog.

Cumulation between PL1 and SG



(→ := ‘*made Abe type*’-relation)

Cumulation between PL1 and PL2



(→ := ‘*made Abe*’-relation)

Claims:

- Flattening examples require the lexical source because of the SG flattening.
- Since the lexical source captures the PL flattening as well, the PL flattening does not support the need of the plural projection.
- We should address cumulations with a singular DP when addressing an analysis of cumulativity.

Road map:
Section 2: The plural projection captures the PL flattening.
Section 3: The plural projection does not capture the SG flattening.
Section 4: The lexical source captures the SG flattening.
Section 5: The lexical source captures the PL flattening for free.
Section 6: Conclusion

2 The plural projection captures the PL flattening.

Cross-categorial plurality: Schmitt (2019) assumes that not only individuals but also expressions with other semantic types denote *pluralities/sums*.

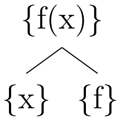
(11) a. $[\![\text{the two boys}]\!] = \text{boy1+boy2}$
b. $[\![\text{sing and dance}]\!] = \textsc{sing}+\textsc{dance}$ $(\textsc{sing} := \lambda x_e.\text{x sings})$
c. $[\![\text{This boy sings and that boy dances}]\!] = \textsc{sing}(\text{boy1})+\textsc{dance}(\text{boy2})$

In addition, she assumes that all lexical items are assigned as their denotations *plural sets*, namely sets involving an element or elements in the relevant domain.

(12) a. $[\![\text{the two boys}]\!] = \{\text{boy1+boy2}\}$
b. $[\![\text{sing and dance}]\!] = \{\textsc{sing}+\textsc{dance}\}$
c. $[\![\text{This boy sings and that boy dances}]\!] = \{\textsc{sing}(\text{boy1})+\textsc{dance}(\text{boy2})\}$

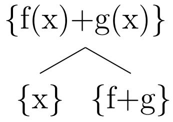
Cumulative compositional rule: Schmitt (2019) proposes a cumulative compositional rule for the composition of plural sets, whose effect is exemplified below (f and g are functions, and x and y are their arguments).

(13) **SG-SG**



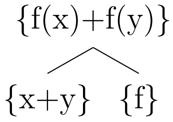
e.g., Abe sings.

(14) **SG-PL**



e.g., Abe sings and dances.

(15) **PL-SG**



e.g., Abe and Bea sing.

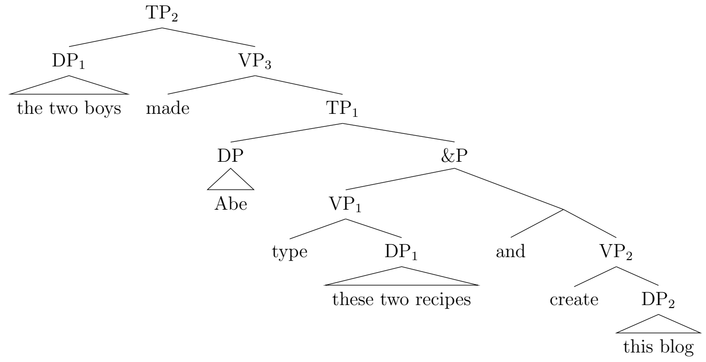
- These are effectively the results of a point-wise functional application.

The derivation of the PL flattening:

(16) [Scenario: Boy1 made Abe type recipe1. Boy2 made Abe type recipe2 and create this blog.]

[The two boys]$_{PL1}$ made Abe [type [these two recipes]$_{PL3}$ and create this blog]$_{PL2}$.

(adapted from Schmitt 2019, 27)



- $[\![\text{VP}_3]\!] = \{\textsc{made}(\textsc{type}(\text{recipe1})(\text{abe}))+\textsc{made}(\textsc{type}(\text{recipe2})(\text{abe}))+\textsc{made}(\textsc{create}(\text{blog})(\text{abe}))\}$
- $[\![\text{DP}_1]\!] = \{\text{boy1+boy2}\}$

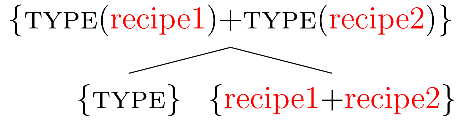
The cumulative compositional rule enables VP$_3$ and DP$_1$ to cumulatively compose.

3 The plural projection does *not* capture the SG flattening.

Cumulative compositional rule treats plurals and singulars differently:

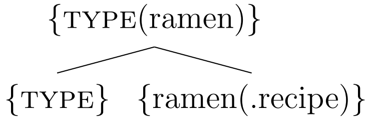
(17) **Plurals**

Correct derivation:

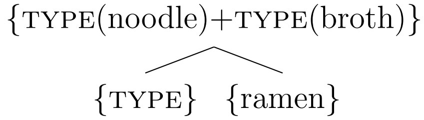


(18) **Singulars**

a. **Correct derivation:**



b. **Incorrect derivation:**



The plural projection undergenerates: This difference results in the plural projection *not* being able to capture the SG flattening.[1]

(19) [Scenario: There is a ramen recipe that consists of two recipes: a recipe about how to make noodle, and a recipe about how to make broth. Boy1 made Abe type the noodle recipe. Boy2 made Abe type the broth recipe, and create a blog.]

[The two boys]$_{PL1}$ made Abe [type **[the ramen recipe]**$_{SG}$ and create this blog]$_{PL2}$.

- The plural projection predicts that the sentence involves the cumulation between: {boy1+boy2} - {MADE(TYPE(**ramen**)(abe))+MADE(CREATE(blog)(abe))}

Singulars denote singularities: This observation is based on the assumption that *the ramen recipe* denotes {ramen} and not pluralities like {noodle+broth}. This assumption is supported by (20).

(20) a. These two recipes are completely correct and completely wrong.
b. # The ramen recipe is completely correct and completely wrong.
(adapted from Paillé 2020)

- If *the ramen recipe* denotes {noodle+broth}, then (20b) should sound as natural as (20a).

4 The lexical source captures the SG flattening

This section demonstrates how the lexical source can capture the SG flattening step by step.

Road map:
Section 4.1: The lexical source captures cumulations with a singular DP.
[The two boys]$_{PL1}$ typed **[the ramen recipe]**$_{SG}$.

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[1] In more general, the plural projection does not captures cumulations with mereological parts of singulars.

(1) [Scenario: There are 50 ramen recipes that consists of the part about how to make noodle and the part about how to make broth. Boy1 typed the noodle parts. Boy2 typed the broth parts.]

The two boys typed the 50 ramen recipes.

Section 4.2: The lexical source captures cumulations with a non-Boolean *and*.
[The two boys]$_{PL1}$ **typed this ramen recipe and created this blog**.

Section 4.3: The lexical source captures cumulations in causative sentences.
[The two boys]$_{PL1}$ **made** Abe [type [the ramen recipe]$_{SG}$.

Section 4.4: The lexical source captures the SG flattening.
[The two boys]$_{PL1}$ made Abe [type [the ramen recipe]$_{SG}$ and create this blog]$_{PL2}$.

4.1 The lexical source captures cumulations with a singular DP.

Ontology: I assume that the domains of DPs and events involve both singularities and pluralities.

(21) a. $D_e = \{\{\text{abe}\}, \{\text{bert}\}, ..., \{\text{abe,bert}\}, ...\}$
b. $D_s = \{\{e_1\}, \{e_2\}, ..., \{e_1,e_2\}, ...\}$

Unlike Schmitt (2019), I assume that the domain of predicates like *type* involve only singularities.

(22) $D_{eest} = \{\text{TYPE, CREATE, ...}\}$,
where $\text{TYPE} = \lambda\chi_e.\lambda\psi_e.\lambda\varepsilon_s.\text{type'}_\varepsilon(\psi,\chi)$ (χ, ψ and ε: a set of individuals/events)

While I represent plurality with *non*-singleton sets, I represent the mereological structures of individuals with sums, if necessary.[2]

(23) $[\![\text{the ramen recipe}]\!] = \{\text{ramen}\} = \{\text{noodle+broth}\} \neq \{\text{noodle, broth}\}$[3]

Event-based semantics: Transitive verbs denote functions of type <e, <e, st>>.

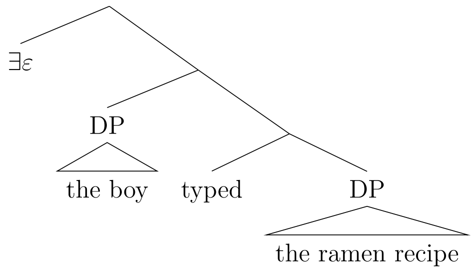
(24) $[\![\text{type}]\!] = \lambda\chi_e.\lambda\psi_e.\lambda\varepsilon_s.\text{type'}_\varepsilon(\psi, \chi)$

Sentences denote a function from events to truth values.

(25) a. The boy typed the ramen recipe.
b. $\lambda\varepsilon_s.\text{type'}_\varepsilon(\{\text{boy1}\},\{\text{ramen}\})$

To enable sentences to denote a truth value, I assume a phonologically null operator '$\exists\varepsilon$' in sentences.

(26) a. The boy typed the ramen recipe.



b. $[\![(26)]\!] = 1$ iff $\exists\varepsilon.\text{type'}_\varepsilon(\{\text{boy1}\},\{\text{ramen}\})$
In other words, (26) is true iff the extension of *typed* contains <{boy1},{ramen}>

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[2] The distinction between the singularity/plurality and mereological structure is an important assumption to capture sentences with a predicate conjunction like (20a-b). See Appendix D.

[3] This is based on the assumption that the referent of *the ramen recipe* consists of a noodle recipe and broth recipe.

Cumulation with a singular DP:

(27) [Scenario: There is a ramen recipe that consists of a noodle recipe and a broth recipe. Boy1 typed the noodle recipe, and boy2 typed the broth recipe.]

[The two boys]$_{PL}$ typed [the ramen recipe]$_{SG}$.

- $[\![(27)]\!] = 1$ iff $\exists\varepsilon.[\text{type'}_{\varepsilon}(\{\text{boy1,boy2}\},\{\text{ramen}\})]$

Does the extension of *type* contain <{boy1,boy2}, {ramen}>? The answer is 'yes' if we assume two things. First, in the given scenario, there are two typing events.

(28) **Scenario:**

a. typed'$_{\{e_1\}}$({boy1},{noodle})
b. typed'$_{\{e_2\}}$({boy2},{broth})
c. So the extension of *typed* looks like: {<{boy1},{noodle}>, <{boy2},{broth}>, ...}

Following Krifka (1989) among others, I assume that the following inference goes through.

(29) **Cumulative inference**
type'$_{\{e_1\}}$({boy1},{noodle}) & type'$_{\{e_2\}}$({boy2},{broth}) →
type'$_{\{e_1,e_2\}}$({boy1,boy2},{noodle,broth})

I also assume that the following inference goes through.[4]

(30) **Parts-whole inference**
type'$_{\{e_1,e_2\}}$({boy1,boy2},{noodle,broth}) & {noodle+broth} = {ramen} →
type'$_{\{e_1,e_2\}}$({boy1,boy2},**{ramen}**)

- "=" is the equivalence that holds in the given scenario.

{e_1,e_2} in (30) serves as a witness for the existence claim in (27).

4.2 The lexical source captures cumulations with a non-Boolean *and*.

This section explains how the lexical source captures cumulations with a non-Boolean *and*.[5]

(31) [Scenario: Boy1 typed this ramen recipe and Boy2 created this blog.]

The two boys typed this ramen recipe and created this blog.

- $[\![\text{and}]\!] = \lambda f_{<e,st>}.\lambda g_{<e,st>}.\lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \;\&\; \varepsilon = \varepsilon_1\cup\varepsilon_2 \;\&\; f(\chi_1)(\varepsilon_1) \;\&\; g(\chi_2)(\varepsilon_2)]$
(c.f., Lasersohn 1995)
- $[\![(31)]\!] = 1$ iff $\exists\chi_1,\chi_2,\varepsilon,\varepsilon_1,\varepsilon_2[\{\text{boy1,boy2}\} = \chi_1\cup\chi_2 \;\&\; \varepsilon = \varepsilon_1\cup\varepsilon_2 \;\&$
$\text{typed'}_{\varepsilon_1}(\chi_1,\{\text{ramen}\}) \;\&\; \text{created'}_{\varepsilon_2}(\chi_2,\{\text{blog}\})]$

In the given scenario, there are two events:

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[4] It remains to be seen whether the parts-whole inference holds for any predicates. See Appendix B for some data that may suggest that the inference does not go through with some predicates.

[5] See Appendix C for a more detailed derivation of (31).

(32) a. $\text{typed'}_{\{e_1\}}(\{\text{boy1}\},\{\text{ramen}\})$

b. $\text{created'}_{\{e_2\}}(\{\text{boy2}\},\{\text{blog}\})$

c. $\{e_1\}\cup\{e_2\} = \{e_1,e_2\}$

$\{e_1\}$, $\{e_2\}$, and $\{e_1,e_2\}$ serve as witnesses for the existence claim in (31).

4.3 The lexical source captures cumulations in causative sentences.

This section demonstrates that the lexical source captures cumulations in causative sentences.

(33) [Scenario: There is a ramen recipe that consists of a noodle recipe and a broth recipe. Boy1 made Abe type the noodle recipe. Boy2 made Abe type the broth recipe.]

[The two boys]$_{PL1}$ made Abe type [the ramen recipe]$_{SG}$.

- $[\![\text{made}]\!] = \lambda\rho_{st}.\lambda\chi.\lambda\varepsilon_s.\exists\varepsilon'.[\text{made}_\varepsilon(\chi,\varepsilon') \ \&\ \rho(\varepsilon')]$
  - *made* has an effect of linking a causing event ε with a caused event ε'
- $[\![(33)]\!] = 1$ iff $\exists\varepsilon,\varepsilon'.[\text{made}_\varepsilon(\{\text{boy1},\text{boy2}\},\varepsilon') \&\ \text{type'}_{\varepsilon'}(\{\text{abe}\},\{\text{ramen}\})]$

In the given scenario, the following events exist.

(34) **Scenario:**

a. $\text{type'}_{\{e'_1\}}(\{\text{abe}\},\{\text{noodle}\})$

b. $\text{type'}_{\{e'_2\}}(\{\text{abe}\},\{\text{broth}\})$

c. $\text{made'}_{\{e_1\}}(\{\text{boy1}\},\{e'_1\})$

d. $\text{made'}_{\{e_2\}}(\{\text{boy2}\},\{e'_2\})$

(35) **After the cumulative/parts-whole inferences**

a. (34a) $\wedge$ (34b) $\rightarrow$ $\text{type'}_{\{e'_1,e'_2\}}(\{\text{abe}\},\{\text{noodle},\text{broth}\})$ **Cumulative inference**

b. (35a) $\wedge$ $\{\text{noodle}+\text{broth}\} = \{\text{ramen}\} \rightarrow \text{type'}_{\{e'_1,e'_2\}}(\{\text{abe}\},\{\text{ramen}\})$ **Parts-whole inference**

c. (34c) $\wedge$ (34d) $\rightarrow$ $\text{made'}_{\{e_1,e_2\}}(\{\text{boy1},\text{boy2}\},\{e'_1,e'_2\})$ **Cumulative inference**

$\{e_1,e_2\}$ and $\{e'_1,e'_2\}$ in (35) serve as witnesses for the existence claims of ε and ε' in (33), respectively.

4.4 The lexical source captures the SG flattening.

Without any further assumption, the lexical source can capture the SG flattening.

(36) [The two boys]$_{PL1}$ made Abe [type **[the ramen recipe]**$_{SG}$ and create this blog]$_{PL2}$. [6]

$[\![(36)]\!] = 1$ iff $\exists\varepsilon,\varepsilon'.[\text{made'}_\varepsilon(\{\text{boy1},\text{boy2}\},\varepsilon')\ \&$

$\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\varepsilon' = \varepsilon'_1\cup\varepsilon'_2\ \&\ \text{create'}_{\varepsilon'_1}(\{\text{abe}\},\{\text{blog}\})\ \&\ \text{type'}_{\varepsilon'_2}(\{\text{abe}\},\{\text{ramen}\})]]$

In the given scenario, the following events exist.

(37) **Scenario:**

a. $\text{type'}_{\{e'_1\}}(\{\text{abe}\}\{\text{noodle}\})$

b. $\text{type'}_{\{e'_2\}}(\{\text{abe}\}\{\text{broth}\})$

c. $\text{create'}_{\{e'_3\}}(\{\text{abe}\}\{\text{blog}\})$

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[6] For a more detailed derivation of the sentence, see Appendix F.

d. $\text{made'}_{\{e_1\}}(\{\text{boy1}\},\{\text{e'}_1\})$
e. $\text{made'}_{\{e_2\}}(\{\text{boy2}\},\{\text{e'}_2\})$
f. $\text{made'}_{\{e_3\}}(\{\text{boy2}\},\{\text{e'}_3\})$

(38) **After the cumulative/parts-whole inference:**

a. $(37a) \wedge (37b) \rightarrow \text{type'}_{\{e'_1,e'_2\}}(\{\text{abe}\}\{\text{noodle,broth}\})$ **Cumulative inference**
b. $(38a) \wedge \{\text{noodle+broth}\} = \{\text{ramen}\} \rightarrow \text{type'}_{\{e'_1,e'_2\}}(\{\text{abe}\}\{\text{ramen}\})$ **Parts-whole inference**
c. $(37d) \wedge (37e) \wedge (37f) \rightarrow \text{made'}_{\{e_1,e_2,e_3\}}(\{\text{boy1,boy2}\},\{\text{e'}_1,\text{e'}_2,\text{e'}_3\})$ **Cumulative inference**

$\{e_1,e_2,e_3\}$, $\{\text{e'}_3\}$, and $\{\text{e'}_1,\text{e'}_2\}$ in (37-38) serve as witnesses for the existence claims of ε and $\varepsilon\text{'}_1$ and $\varepsilon\text{'}_2$ in (36).

5 The lexical source captures the PL flattening *for free*

Without any further assumption, the lexical source can capture the PL flattening.

(39) $[\text{The two boys}]_{PL1}$ made Abe [type **[these two recipes]**$_{PL3}$ and create this blog$]_{PL2}$.
$[\![(39)]\!] = 1$ iff $\exists\varepsilon,\varepsilon\text{'}.[\text{made'}_\varepsilon(\{\text{boy1,boy2}\},\varepsilon\text{'})$ &
$\exists\chi_1,\chi_2,\varepsilon\text{'}_1,\varepsilon\text{'}_2[\varepsilon\text{'}=\varepsilon\text{'}_1\cup\varepsilon\text{'}_2\ \&\text{create'}_{\varepsilon'_1}(\{\text{abe}\},\{\text{blog}\})\&\ \text{type'}_{\varepsilon'_2}(\{\text{abe}\},\{\text{recipe1,recipe2}\})]]$

- $\{\text{e'}_1,\text{e'}_2\}$ in (38a) (instead of (38b)) serves as a witness for the existence claim of $\varepsilon\text{'}_2$ in (39).
- The truth of the PL flattening in (39) can be confirmed with *one less premise* (i.e., the parts-whole inference in (38b)) than the SG flattening in (36).

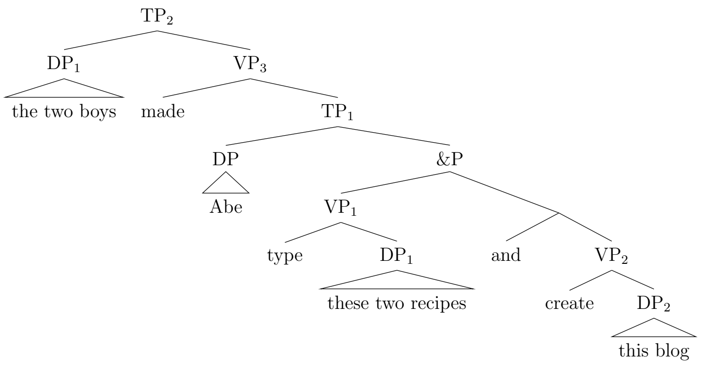
6 Conclusion

- **Observation:** The plural projection does *not* capture the SG flattening. (see Section3)
- **Observation:** To capture the SG flattening, we need a source of cumulations like the lexical source (e.g., Krifka 1989, Kratzer 2007), which does not need multiple *plurals*. (see Section4)
- **Observation:** Such a cumulative source also captures the PL flattening *for free*. (see Section5)
- **Claim:** The most parsimonious analysis to capture flattening examples is the one with only the lexical source.
- The PL flattening does not support the need of the plural projection.
- **Claim:** It is important to take account of cumulations with a singular DP when we address an analysis of cumulativity.

A Appendix: Derivation of (16) with the plural projection

(40) [Scenario: Boy1 made Abe type recipe1. Boy2 made Abe type recipe2 and create this blog.]

$[\text{The two boys}]_{PL1}$ made Abe [type [these two recipes$]_{PL3}$ and create this blog$]_{PL2}$.
(adapted from Schmitt 2019, 27)



- $[\![\mathrm{VP}_1]\!] = \{\textsc{type}(\mathrm{recipe1})+\textsc{type}(\mathrm{recipe2})\}$
- $[\![\mathrm{VP}_2]\!] = \{\textsc{create}(\mathrm{blog})\}$
- $[\![\mathrm{and}]\!] = \lambda \mathrm{X}_{\{a\}}.\lambda \mathrm{Y}_{\{a\}}.\{\mathrm{x}+\mathrm{y}\text{: } \mathrm{x}\in\mathrm{X},\ \mathrm{y} \in\mathrm{Y}\}$
- $[\![\&\mathrm{P}]\!] = \{\textsc{type}(\mathrm{recipe1})+\textsc{type}(\mathrm{recipe2})+\textsc{create}(\mathrm{blog})\}$
- $[\![\mathrm{VP}_3]\!] = \{\textsc{made}(\textsc{type}(\mathrm{recipe1})(\mathrm{abe}))+\textsc{made}(\textsc{type}(\mathrm{recipe2})(\mathrm{abe}))+\textsc{made}(\textsc{create}(\mathrm{blog})(\mathrm{abe}))\}$
  $= \{\mathrm{f}+\mathrm{g}+\mathrm{h}\}$
- $[\![\mathrm{DP}_1]\!] = \{\mathrm{boy1}+\mathrm{boy2}\}$
- $[\![\mathrm{TP}_1]\!] = \{\mathbf{f(boy1)+g(boy2)+h(boy2)}, ..., \mathrm{f(boy1)+g(boy1)+h(boy1)+f(boy2)+g(boy2)+h(boy2)}\}$
- $[\![\mathrm{TP}_2]\!]$ says: (16) is true iff at least one of the pluralities in $[\![\mathrm{TP}_1]\!]$ is true.

B Appendix: Cumulations with a singular DP sometimes seem to be disallowed.

While cumulations in (41) are both available, the cumulations with a singular DP seem to be available only in (42a).

(41) [Scenario: My dissertation consists of three chapters. Professor1 read/accepted Chapter 1 and 2 and Professor2 read/accepted Chapter 3.]

a. The two professors **read** the three chapters.
b. The two professors **accepted** the three chapters.

(42) **Cumulation with a singular DP**
[Scenario: same as (41)]

a. The two professors **proved** my dissertation.
b. $^{??}$The two professors **accepted** my dissertation.

The contrast between (42a-b) seems *prima facie* to suggest that some predicates like *accepted* do not allow the parts-whole inference. However, such an analysis is not correct because *accepted* does allow the parts-whole inference in some scenarios:

(43) [Scenario: My dissertation consists of three chapters. It is considered to be completed if each of my advisors, Prof1 and Prof2, accepts the part (s)he is supposed to evaluate. Prof1 is supposed to evaluate only Chapter 1 and 2 and accepted them, and Prof2 is supposed to evaluate only Chapter 3 and accepted it.]

Abe: Did you finished your dissertation?
Bert: Yeah, yesterday, the two professors (in my committee) **accepted** my dissertation.

So it could be the case that any predicates allow the parts-whole inference in principle and that some sentences like (42) seem to disallow the sub-atomic cumulations only because the relevant predicates like *accept* is usually used to derive distributive interpretations.[7]

Other examples like (42b) are the following.

(44) [Scenario: There is a disease called Disease X, and the preventive treatment for this disease is ready; that is to take three different kinds of shots, Shot1, Shot2, and Shot3. The order of taking those shots does not matter. Boy1 finished taking Shot1 and Shot2, and boy2 finished taking Shot3.]
a. The two boys finished the three shots.
b. #The two boys finished the Disease X treatment.

(45) [Scenario: There is a course in which students' grades are determined only by their performance of two mid-terms and one final exam. Crucially, in order to pass the course, students have to pass all of the exams. Boy1 passed the two midterm and boy2 passed the final.]
a. The two boys passed the three exams.
b. #The two boys passed the course.

(46) [Scenario: There is a ramen recipe which consists of a noodle recipe, broth recipe, and a char siu pork recipe. Each of the three recipe is two-page long.]
a. These three recipes are two-page long.
b. #This ramen recipe is two-page long.

These examples seem to disallow the cumulations with a singular DP more clearly than (42b). But those sentences also seem to allow such cumulations under an appropriate scenario, as shown in (47) for (44).

(47) [Scenario: A disease called Disease X is spreading and three different kinds of shots were invented as the preventive treatment for it. For some reason it is said that everyone does not have to take all kinds of the shots; instead, it is advised that for each of the three shots, each family has at least one member who has taken it. Abe and Bert are brothers. Abe took Shot 1 and Shot2, and Bert finished Shot3.]
The two boys finished the Disease X treatment.

Also, some sentences that clearly allow cumulations with a singular DP do not allow them under some scenarios. For example, (48a) is true in (48b) but not in (48c).

(48) a. [The two boys]$_{PL}$ prepared [a hamburger combo.]$_{SG}$
b. [Scenario: Boy1 and boy2 are working at a hamburger shop. A customer ordered a hamburger combo which consists of a hamburger and french fries. For the hamburger combo, Boy1 prepared a hamburger, and boy2 prepared french fries.]
c. [Scenario: Boy1, boy2, and girl1 are working at a hamburger shop. Boy1 just prepared a hamburger for a customer. Boy2 just prepared french fries for a different customer. Girl1 prepared a hamburger combo for another customer.]

But it remains to be seen whether all predicates allow cumulations with a singular DP in principle.

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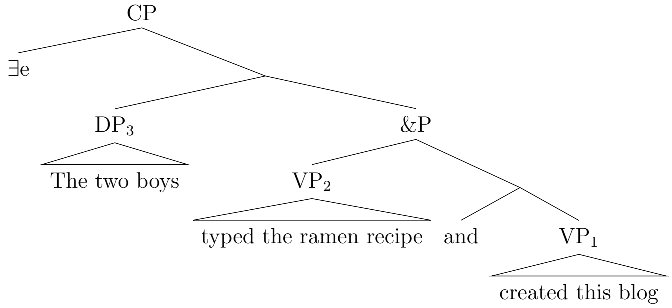
[7] Under this analysis, it can be assumed that (42b) sounds degraded unlike (41b) because the singularity of *my dissertation* makes it difficult for the listener to access the part structure of the DP.

C Appendix: Derivation of (31) with the lexical source

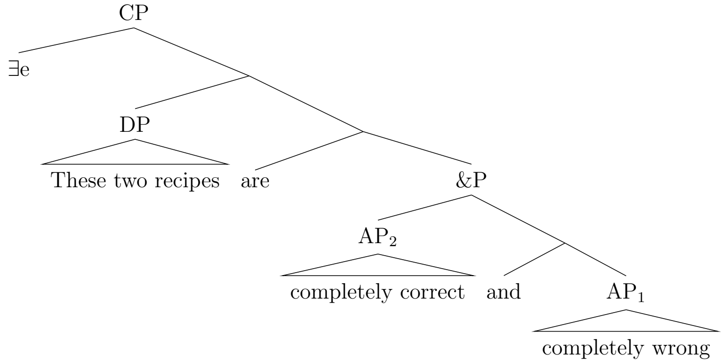
(49) [Scenario: Boy1 typed this ramen recipe and Boy2 created this blog.]
The two boys typed this ramen recipe and created this blog.



- $\llbracket \mathrm{VP}_1 \rrbracket = \lambda\chi_e.\lambda\varepsilon_s.\mathrm{typed'}_\varepsilon(\chi,\{\mathrm{ramen}\})$
- $\llbracket \mathrm{VP}_2 \rrbracket = \lambda\chi_e.\lambda\varepsilon_s.\mathrm{created'}_\varepsilon(\chi,\{\mathrm{blog}\})$
- $\llbracket \mathrm{and} \rrbracket = \lambda \mathrm{f}_{<e,st>}.\lambda \mathrm{g}_{<e,st>}.\lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \& \ \mathrm{f}(\chi_1)(\varepsilon_1) \ \& \ \mathrm{g}(\chi_2)(\varepsilon_2)]$
(c.f., Lasersohn 1995)
- $\llbracket \&\mathrm{P} \rrbracket = \lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \& \ \llbracket \mathrm{VP}_1 \rrbracket(\chi_1)(\varepsilon_1) \ \& \ \llbracket \mathrm{VP}_2 \rrbracket(\chi_2)(\varepsilon_2)]$
$= \lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \&$
$\mathrm{typed'}_{\varepsilon_1}(\chi_1,\{\mathrm{ramen}\}) \ \& \ \mathrm{created'}_{\varepsilon_2}(\chi_2,\{\mathrm{blog}\})]$
- $\llbracket \mathrm{CP} \rrbracket = 1$ iff $\exists\chi_1,\chi_2,\varepsilon,\varepsilon_1,\varepsilon_2[\{\mathrm{boy1},\mathrm{boy2}\} = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \&$
$\mathrm{typed'}_{\varepsilon_1}(\chi_1,\{\mathrm{ramen}\}) \ \& \ \mathrm{created'}_{\varepsilon_2}(\chi_2,\{\mathrm{blog}\})]$

D Appendix: The derivation of (20) with the lexical source

(50) These two recipes are completely correct and completely wrong.
(adapted from Paillé (2020))



- $\llbracket \mathrm{AP}_1 \rrbracket = \lambda\chi_e.\lambda\varepsilon_s.\mathrm{cw'}_\varepsilon(\chi)$
- $\llbracket \mathrm{AP}_2 \rrbracket = \lambda\chi_e.\lambda\varepsilon_s.\mathrm{cc'}_\varepsilon(\chi)$
- $\llbracket \mathrm{and} \rrbracket = \lambda \mathrm{f}_{<e,st>}.\lambda \mathrm{g}_{<e,st>}.\lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \& \ \mathrm{f}(\chi_1)(\varepsilon_1) \ \& \ \mathrm{g}(\chi_2)(\varepsilon_2)]$
- $\llbracket \&\mathrm{P} \rrbracket = \lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \& \ \llbracket \mathrm{AP}_1 \rrbracket(\chi_1)(\varepsilon_1) \ \& \ \llbracket \mathrm{AP}_2 \rrbracket(\chi_2)(\varepsilon_2)]$
$= \lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \& \ \mathrm{cw'}_{\varepsilon_1}(\chi_1) \ \& \ \mathrm{cc'}_{\varepsilon_2}(\chi_2)]$
- $\llbracket \mathrm{CP} \rrbracket = \exists\chi_1,\chi_2,\varepsilon,\varepsilon_1,\varepsilon_2[\{\mathrm{recipe1},\mathrm{recipe2}\} = \chi_1\cup\chi_2 \ \& \ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \& \ \mathrm{cw'}_{\varepsilon_1}(\chi_1) \ \& \ \mathrm{cc'}_{\varepsilon_2}(\chi_2)]$

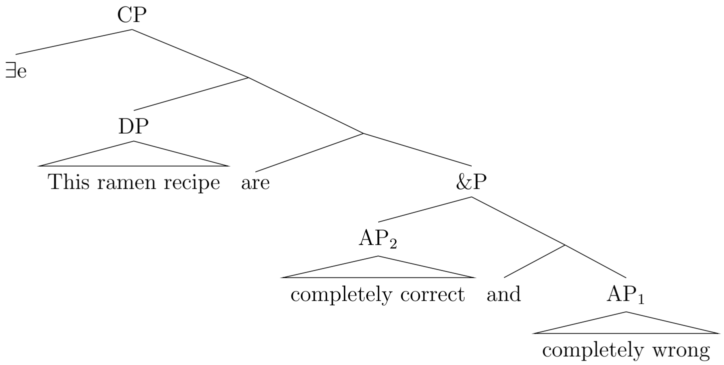
In the given scenario, there are two events:

(51)
a. $\text{cw'}_{\{e1\}}(\{\text{recipe1}\})$
b. $\text{cc'}_{\{e2\}}(\{\text{recipe2}\})$
c. $\{\text{e1}\}\cup\{\text{e2}\} = \{\text{e1,e2}\}$

$\{\text{e1}\}$, $\{\text{e2}\}$, and $\{\text{e1,e2}\}$ serve as witnesses for the existence claim in (50).

Next, consider (52).

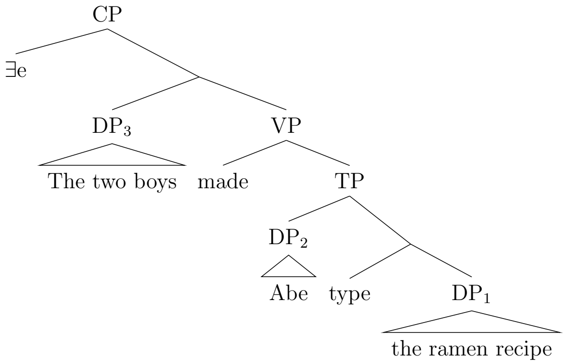
(52) $^{\#}$This ramen recipe is completely correct and completely wrong.
(adapted from Paillé (2020))



- $[\![\text{CP}]\!] = \exists\chi_1,\chi_2,\varepsilon,\varepsilon_1,\varepsilon_2[\{\text{ramen}\} = \chi_1\cup\chi_2 \; \& \; \varepsilon = \varepsilon_1\cup\varepsilon_2 \; \& \; \text{cw'}_{\varepsilon_1}(\chi_1) \; \& \; \text{cc'}_{\varepsilon_2}(\chi_2)]$
- (52) is correctly predicted infelicitous because χ_and χ_2 are both {ramen}, and the sentence means the ramen recipe is both completely correct and completely wrong.

E Derivation of (33) with the lexxical source

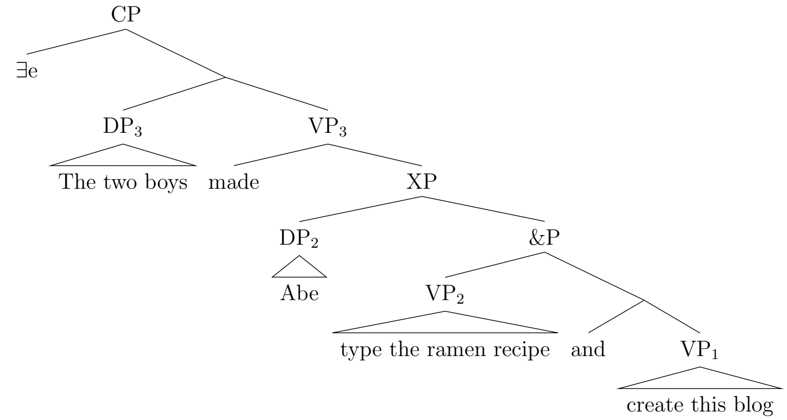
(53) [Scenario: There is a ramen recipe that consists of a noodle recipe and broth recipe. Boy1 made Abe type the noodle recipe. Boy2 made Abe type the broth recipe.]
[The two boys]$_{PL1}$ made Abe type [the ramen recipe]$_{SG}$.



- $[\![\text{XP}]\!] = \lambda\varepsilon_s.\text{type'}_\varepsilon(\{\text{abe}\},\{\text{ramen}\})$
- $[\![\text{made}]\!] = \lambda\rho_{st}.\lambda\chi.\lambda\varepsilon_s.\exists\varepsilon'.[\text{made}_\varepsilon(\chi,\varepsilon') \; \& \; \rho(\varepsilon')]$
- $[\![\text{VP}]\!] = \lambda\chi.\lambda\varepsilon_s.\exists\varepsilon'.[\text{made}_\varepsilon(\chi,\varepsilon') \; \& \; \text{type'}_{\varepsilon'}(\{\text{abe}\},\{\text{ramen}\})]$
- $[\![\text{CP}]\!] = \exists\varepsilon,\varepsilon'.[\text{made}_\varepsilon(\{\text{boy1,boy2}\},\varepsilon')\& \; \text{type'}_{\varepsilon'}(\{\text{abe}\},\{\text{ramen}\})]$

F Appendix: Derivation of (36).

(54) [The two boys]$_{PL1}$ made Abe [type **[the ramen recipe]**$_{SG}$ and create this blog]$_{PL2}$.



- $[\![\text{VP}_1]\!] = \lambda\chi_e.\lambda\varepsilon'_s.\text{create'}_{\varepsilon'}(\chi,\{\text{blog}\})$
- $[\![\text{VP}_2]\!] = \lambda\chi_e.\lambda\varepsilon'_s.\text{type'}_{\varepsilon'}(\chi,\{\text{ramen}\})$
- $[\![\text{and}]\!] = \lambda \text{f}_{<e,st>}.\lambda \text{g}_{<e,st>}.\lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \;\&\; \varepsilon = \varepsilon_1\cup\varepsilon_2 \;\&\; \text{f}(\chi_1)(\varepsilon_1) \;\&\; \text{g}(\chi_2)(\varepsilon_2)]$
- $[\![\&\text{P}]\!] = \lambda\chi_e.\lambda\varepsilon'_s.\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\chi = \chi_1\cup\chi_2 \;\&\; \varepsilon' = \varepsilon'_1\cup\varepsilon'_2 \;\&\; [\![\text{VP}_1]\!](\chi_1)(\varepsilon'_1) \;\&\; [\![\text{VP}_2]\!](\chi_2)(\varepsilon'_2)]$
  $= \lambda\chi_e.\lambda\varepsilon'_s.\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\chi = \chi_1\cup\chi_2 \;\&\; \varepsilon' = \varepsilon'_1\cup\varepsilon'_2 \;\&\; \text{create'}_{\varepsilon'_1}(\chi_1,\{\text{blog}\}) \;\&$
  $\text{type'}_{\varepsilon'_2}(\chi_2,\{\text{ramen}\})]$
- $[\![\text{TP}]\!] = \lambda\varepsilon'_s.\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\{\text{abe}\} = \chi_1\cup\chi_2 \;\&\; \varepsilon' = \varepsilon'_1\cup\varepsilon'_2 \;\&\; \text{create'}_{\varepsilon'_1}(\chi_1,\{\text{blog}\}) \;\&$
  $\text{type'}_{\varepsilon'_2}(\chi_2,\{\text{ramen}\})]$
  $= \lambda\varepsilon'_s.\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\varepsilon' = \varepsilon'_1\cup\varepsilon'_2 \;\&\; \text{create'}_{\varepsilon'_1}(\{\text{abe}\},\{\text{blog}\}) \;\&\; \text{type'}_{\varepsilon'_2}(\{\text{abe}\},\{\text{ramen}\})]$
- $[\![\text{made}]\!] = \lambda\rho_{st}.\lambda\chi.\lambda\varepsilon_s.\exists\varepsilon'.[\text{made}_{\varepsilon}(\chi,\varepsilon') \;\&\; \rho(\varepsilon')]$
- $[\![\text{VP}_3]\!] = \lambda\chi.\lambda\varepsilon_s.\exists\varepsilon'.[\text{made'}_{\varepsilon}(\chi,\varepsilon') \;\&$
  $\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\varepsilon' = \varepsilon'_1\cup\varepsilon'_2 \;\&\; \text{cretate'}_{\varepsilon'_1}(\{\text{abe}\},\{\text{blog}\}) \;\&\; \text{type'}_{\varepsilon'_2}(\{\text{abe}\},\{\text{ramen}\})]]$
- $[\![\text{CP}]\!] = \exists\varepsilon,\varepsilon'.[\text{made'}_{\varepsilon}(\{\text{boy1},\text{boy2}\},\varepsilon') \;\&$
  $\exists\chi_1,\chi_2,\varepsilon'_1,\varepsilon'_2[\varepsilon' = \varepsilon'_1\cup\varepsilon'_2 \;\&\; \text{create'}_{\varepsilon'_1}(\{\text{abe}\},\{\text{blog}\}) \;\&\; \text{type'}_{\varepsilon'_2}(\{\text{abe}\},\{\text{ramen}\})]]$

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